WNE Linear Algebra Final Exam

Series A

12 March 2013

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and its series.**

Problem 1.
Let $v_1 = (1,1,0,1), v_2 = (1,-1,1,2), v_3 = (4,2,1,5), v_4 = (0,-2,1,1)$. Set $V = \text{lin}(v_1, v_2, v_3, v_4)$.

a) find a basis of the space V and a system of linear equations which set of solutions is equal to V,
b) let $W_t = \text{lin}((5,-1,t,8))$. For which $t \in \mathbb{R}$ the space V contains W_t, i.e. $W_t \subset V$?

Problem 2.
Let $W \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + 2x_2 - x_3 - x_4 - x_5 = 0 \\ 2x_1 + 4x_2 + x_3 - 6x_5 = 0 \\ 3x_1 + 6x_2 - 2x_3 - x_4 - 7x_5 = 0 \end{cases}$$

a) find dimension of the space W,
b) find a basis $\mathcal{A}$ of W such that the vector $(1,1,0,2,1)$ has all coordinates equal to 1 relative to $\mathcal{A}$.

Problem 3.
Let $\mathcal{A}$ be basis of $\mathbb{R}^3$ and let $\mathcal{C} = \{(1,2),(-1,3)\}$ be a basis of $\mathbb{R}^2$. Let $\mathcal{B}$ be a basis of $\mathbb{R}^2$ such that $M(id)_{\mathcal{B}}^{\mathcal{C}} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. A linear transformation $\varphi : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given by the matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

a) find matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{C}}$,
b) find basis $\mathcal{B}$.

Problem 4.
Let $v_1 = \frac{1}{3}(2,-2,0,1), v_2 = \frac{1}{3}(2,2,1,0)$ and let $w = (0,0,3,6)$.

a) find the orthogonal projection of w onto $\text{lin}(v_1, v_2)$,
b) find $v_3 \in \mathbb{R}^3$ such that v_1, v_2, v_3 is an orthogonal basis of $W = \text{lin}(v_1, v_2, w)$.

Problem 5.

Let $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 2 & -2 & 2 & 6 \\ 3 & 0 & 1 & 4 \end{bmatrix}$.

a) compute $\det A$,

b) let $B = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$ and let $C \in M(2 \times 2; \mathbb{R})$ be a matrix such that $\det(BC^{\mathsf{T}}) = 32$. Compute $\det(C^3)$.

Problem 6.

Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$.

a) find $C \in M(2 \times 2; \mathbb{R})$ and $s \in \mathbb{R}$ such that $C^{-1}AC = \begin{bmatrix} 2 & 0 \\ 0 & s \end{bmatrix}$,

b) compute A^{200}.

Problem 7.

Let $P = (1, 2, 3), Q = (2, 1, 4)$ be points in the affine space $\mathbb{R}^3$.

a) find a parametrization of the line L passing through P and Q and a system of linear equations describing L,

b) find an equation describing plane $H \subset \mathbb{R}^3$ for which Q is the orthogonal projection of P on H.

Problem 8.

Consider the following linear programming problem $x_1 - 2x_2 + x_3 \to \min$ in the standard form with constraints

$$\begin{cases} 2x_1 + x_2 + x_3 + 2x_4 = 2 \\ 2x_1 + x_2 + 3x_4 + x_5 = 4 \end{cases} \text{ and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the basic solutions $\mathcal{B}_1 = \{2, 3\}, \mathcal{B}_2 = \{3, 5\}$ are feasible?

b) solve the above linear programming problem using simplex method.